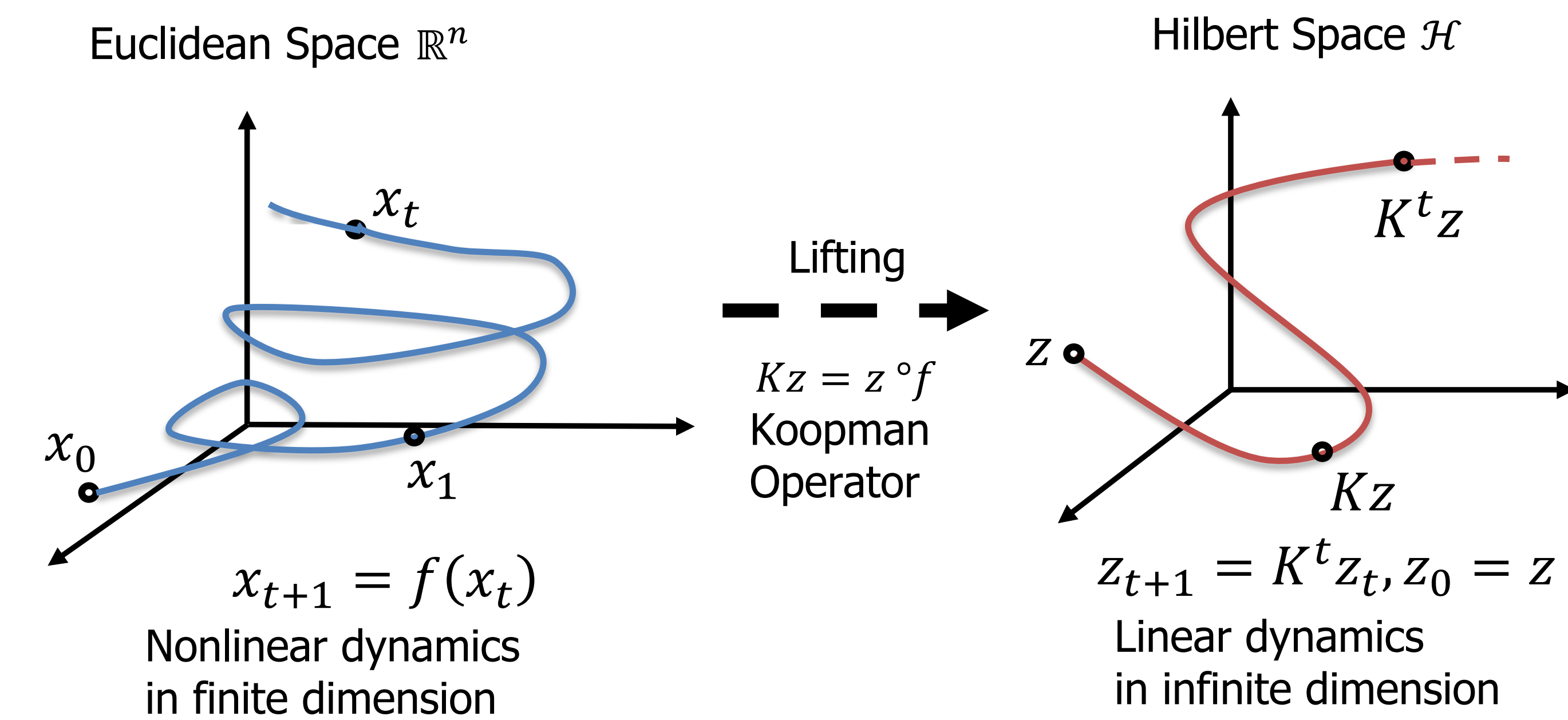


## Motivations

- Finding optimal treatments for wound healing is important
- Challenges in applying Deep Reinforcement Learning (DRL) to biology:
  - Exploration should be constrained: ethical and safety concerns
  - We need guidance that help incrementally increase the performance of target policy
- Wound dynamics are intricate and nonlinear, making mathematical modeling hard
- Getting optimal controls for nonlinear dynamics is difficult
- Theory in linear models is well-developed

**Objective:** Design a DRL-based algorithm for accelerating wound healing while balancing the trade-off between exploration and exploitation

## Learning Linear Representation Example: Koopman Operator Theory



## Challenges:

- The optimization lies in function space, rendering it intractable in practical applications
- It is often hard to account for the effect of inputs and control in nonlinear systems
- Generalizations consider the control effect using deep learning, but still heavily rely on overfitting the models without incrementally and adaptively online learning

## DeepMapper

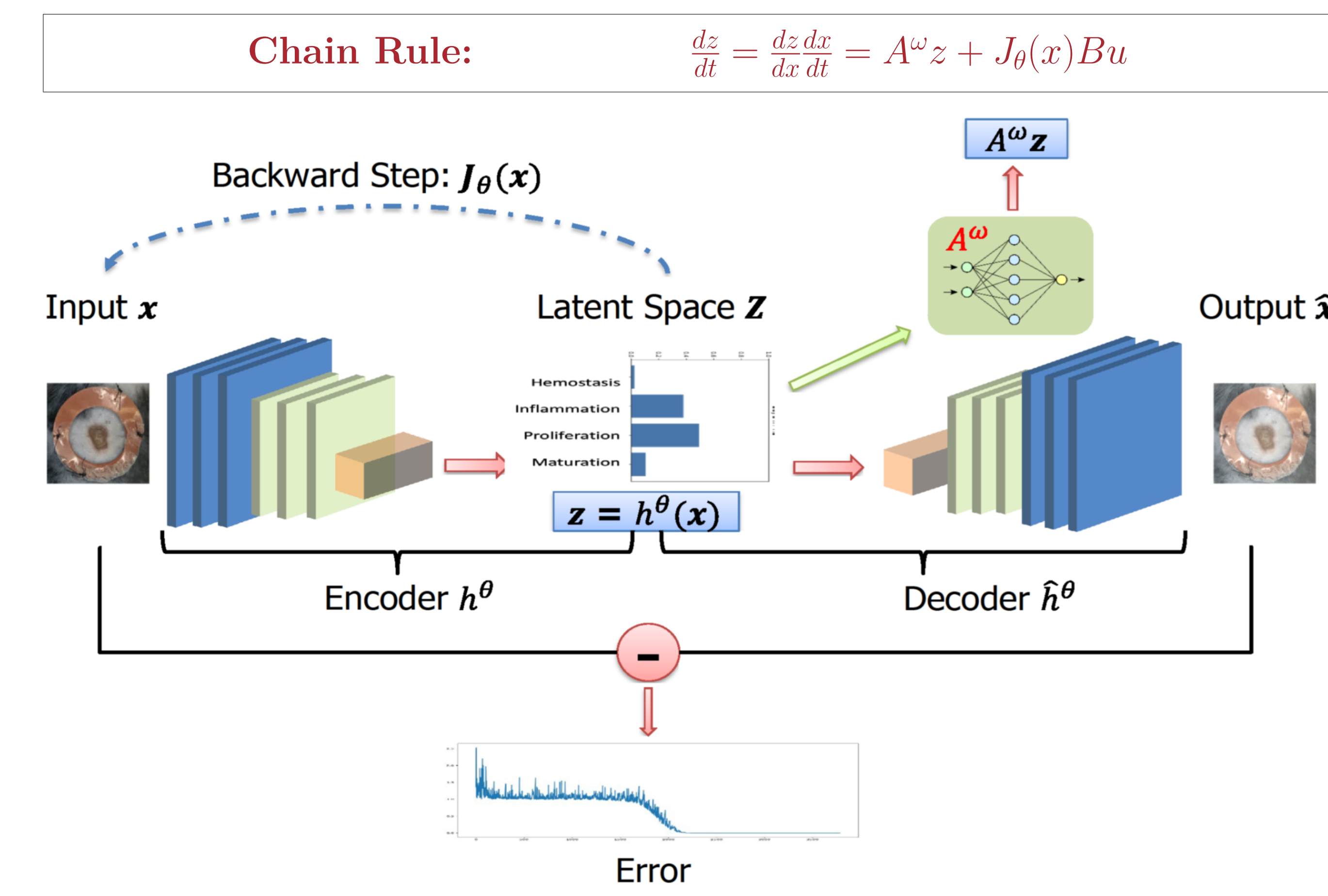
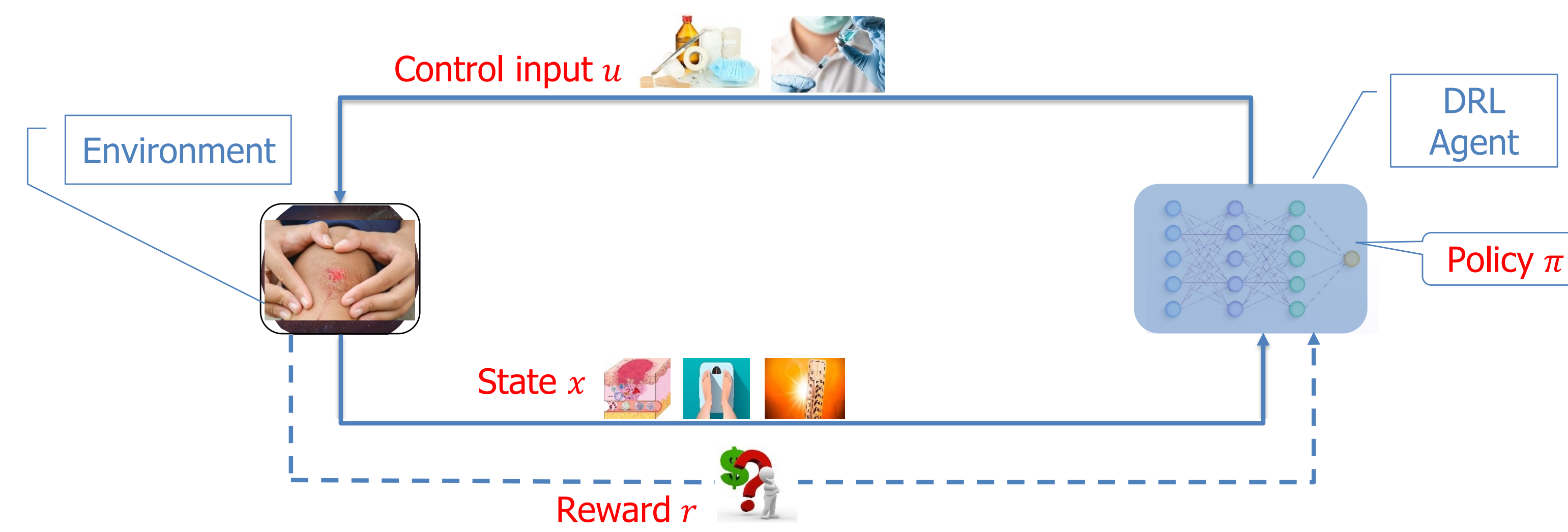


Figure: A self-adaptive linearity abstraction for learning the linear representation of latent space in Autoencoders for nonlinear wound healing dynamics

## Deep Q-Learning

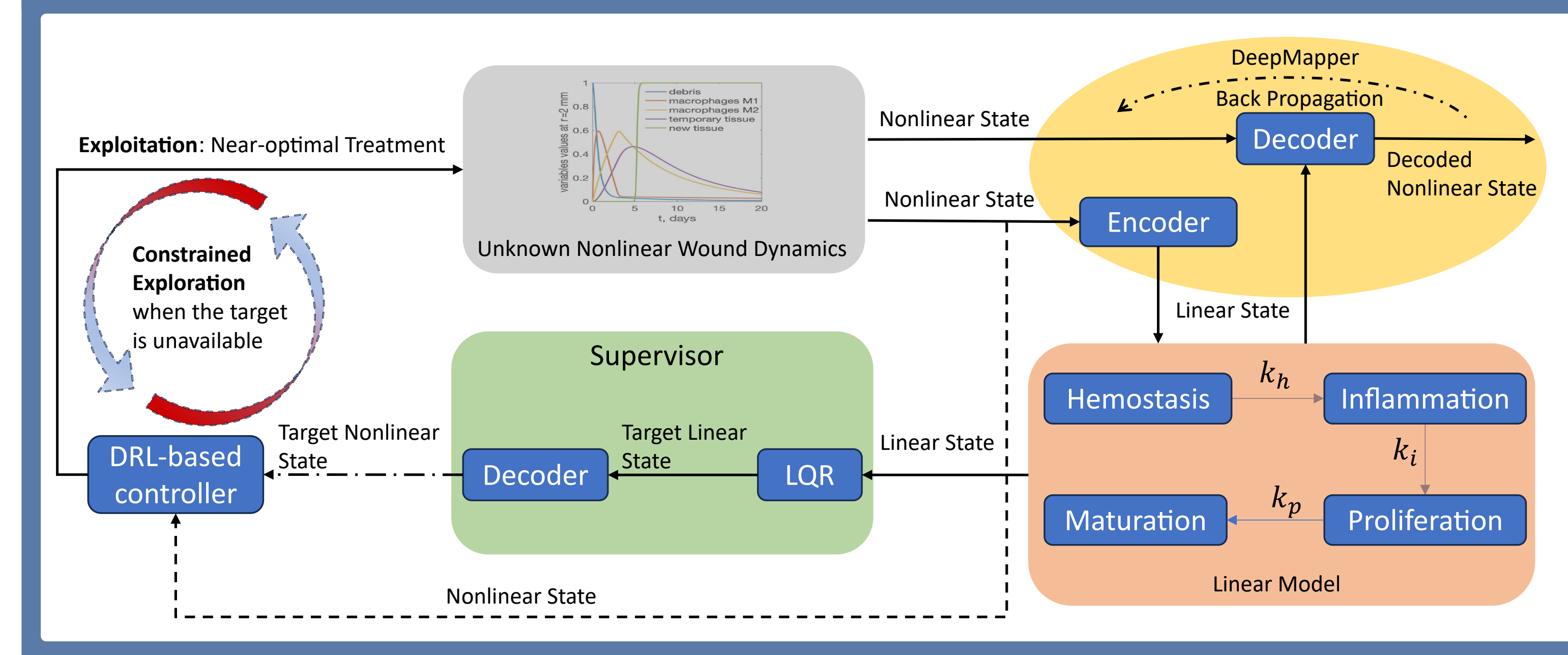


**Goal:** Find a policy  $\pi^\theta$  such that  $\sum_{t=0}^{\infty} \gamma^t r(x_t, u_t)$  is maximized with  $\theta$  the weight of DNNs.

$$\theta_{t+1} = \theta_t + \alpha_{t+1} [r(x_t, u_t) + \bar{Q}^{\theta_{t+1}}(x_{t+1}) - \gamma Q^{\theta_t}(x_t, u_t)] \nabla_{\theta} Q^{\theta_t}(x_t, u_t) |_{\theta=\theta_t}$$

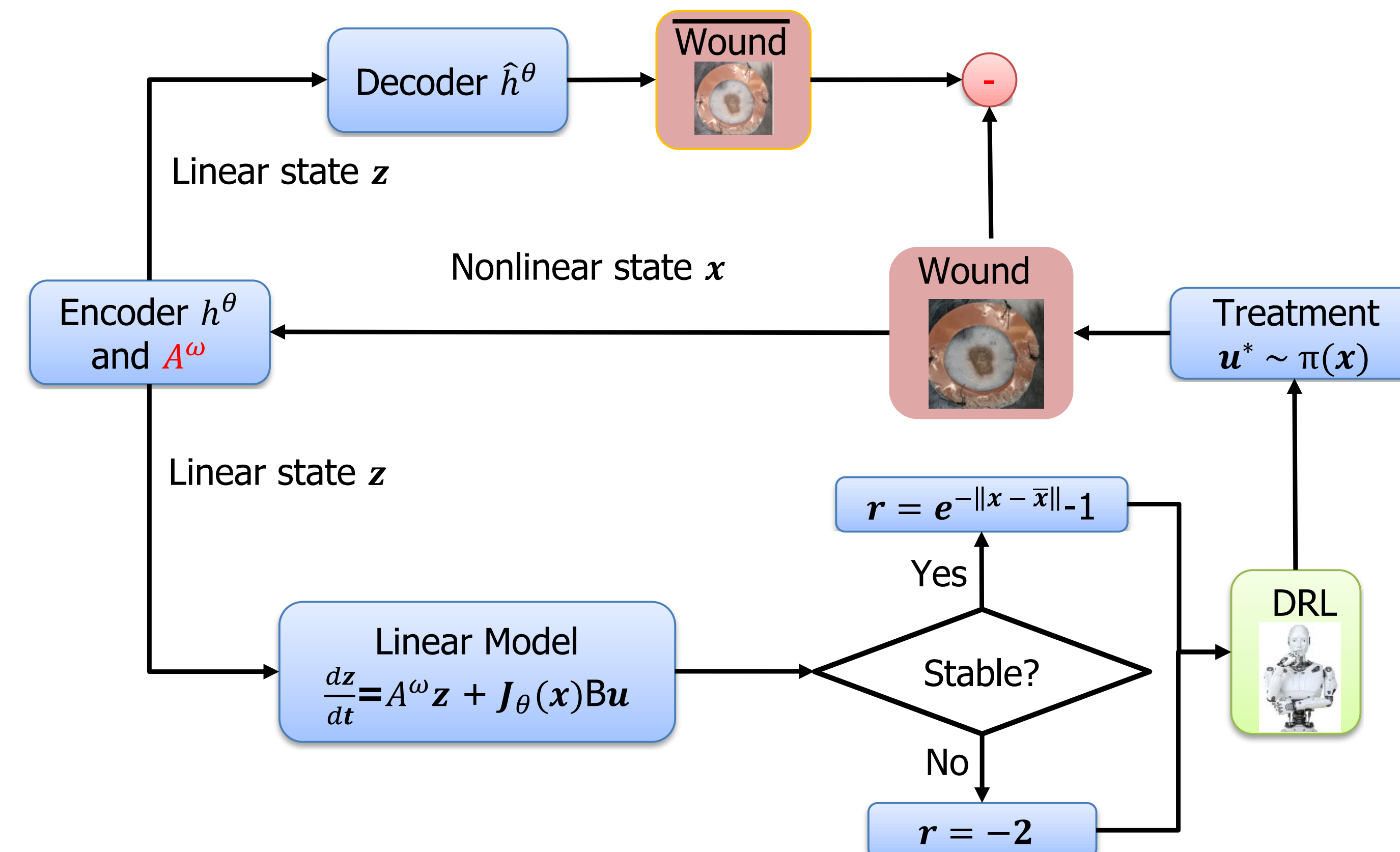
$$\text{where } \pi^\theta(x) \in \arg \max_{u \in U} Q^\theta(x, u), \quad \bar{Q}^\theta(x) := \max_{u \in U} Q^\theta(x, u)$$

## Framework



## DeepMapper: Where Does the Control $u$ Come From

- DRL agent wants to find a policy  $\pi$  that tracks the decoded nonlinear signal of the linear signal that is optimal if it is stable
- DRL agent will get punished for generating control that makes the linear representation unstable
- We have formulated a closed-loop control framework that learns optimal control policy adaptively and incrementally with a safety guide



## Numerical Study: Problem Setup

### Nonlinear Wound dynamics:

$$\text{Debris: } \dot{a} = -am_1$$

$$\text{M1: } \dot{m}_1 = \beta a - \dot{a} - \rho \frac{m_1^q}{k^q + m_1^q} - \gamma_1 m_1 + \tilde{D} \left[ \frac{1}{\tilde{r}} \frac{\partial m_1}{\partial \tilde{r}} + \frac{\partial^2 m_1}{\partial \tilde{r}^2} \right] - um_1$$

$$\text{M2: } \dot{m}_2 = \rho \frac{m_1^q}{k^q + m_1^q} - \gamma_2 m_2 + \tilde{D} \left[ \frac{1}{\tilde{r}} \frac{\partial m_2}{\partial \tilde{r}} + \frac{\partial^2 m_2}{\partial \tilde{r}^2} \right] + um_1$$

$$\text{Temporary Tissue: } \dot{c} = m_2 - \mu c$$

$$\text{New Tissue: } \dot{n} = c \left[ \tilde{\alpha} n(1 - n) + \tilde{D}_n \left[ \frac{1}{\tilde{r}} \frac{\partial n}{\partial \tilde{r}} + \frac{\partial^2 n}{\partial \tilde{r}^2} \right] \right]$$

### Linear Representation:

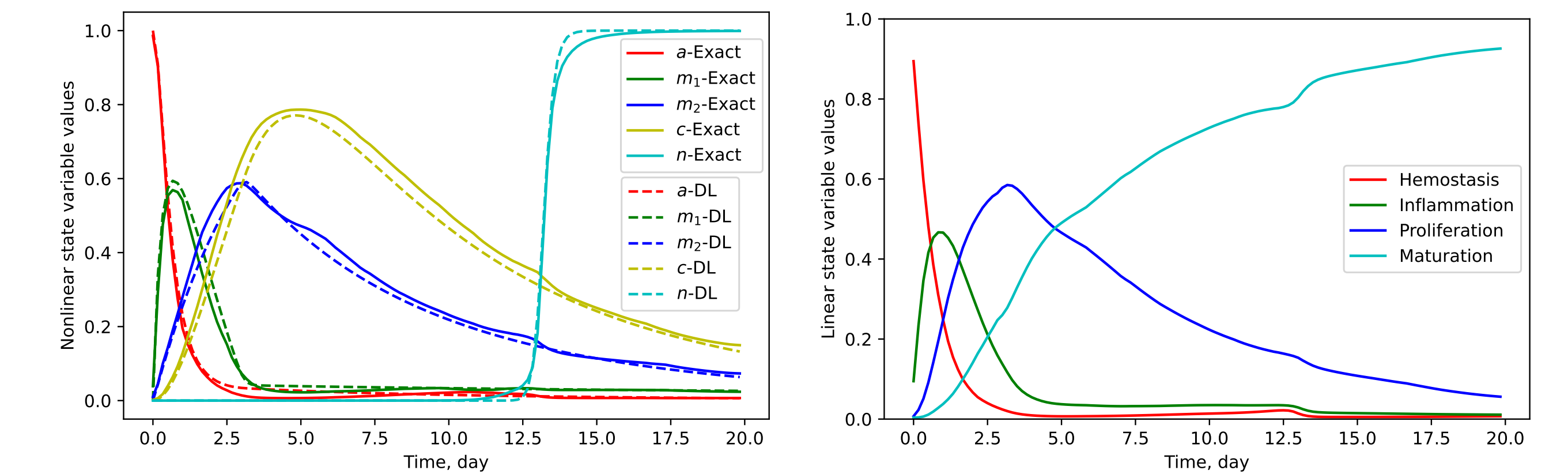
$$\text{Hemostasis: } \dot{H} = -k_h H$$

$$\text{Inflammation: } \dot{I} = k_h H - k_i I$$

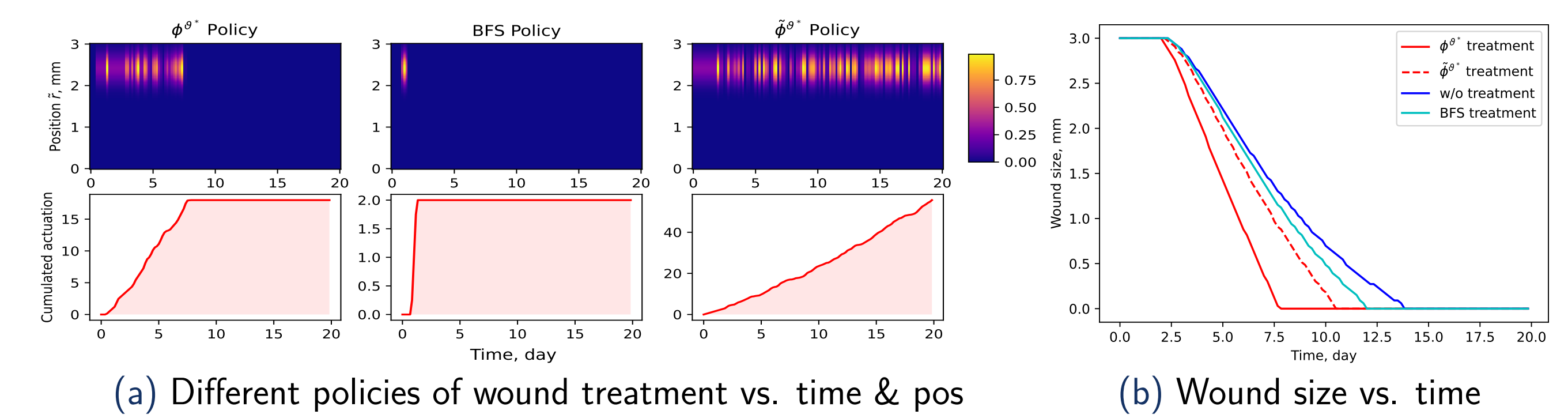
$$\text{Proliferation: } \dot{P} = k_i I - k_p P$$

$$\text{Maturation: } \dot{M} = k_p P$$

## Numerical Study: Performance of DeepMapper



## Numerical Study: Policy Comparison



## Conclusion & Future Work

- A general closed-loop control framework that incorporates deep learning, optimal adaptive control, and reinforcement learning for accelerating wound healing
- The proposed method has successfully reduced the wound healing time by a significant quantity compared to those without any treatment and treatment found by [4], as well as capturing the concerns of safety and less drug usage
- Optimality guarantee from the DRL algorithm [1, 2, 3]?

## References

- [1] Fan Lu, Prashant G. Mehta, Sean P. Meyn and Gergel Neu *Convex Q-learning American Control Conference (ACC)*. Dec. 2021.
- [2] Fan Lu, Prashant G. Mehta, Sean P. Meyn and Gergel Neu *Convex Analytic Theory for Convex Q-Learning Control Conference on Decision and Control (CDC)*. Dec. 2022.
- [3] Fan Lu, and Sean P. Meyn *Convex Q Learning in a Stochastic Environment Control Conference on Decision and Control (CDC)*. Dec. 2023 (To Appear).
- [4] Ksenia Zlobina<sup>1</sup>, Jiahao Xue, and Marcella Gomez *Effective Spatio-Temporal Regimes for Wound Treatment by Way of Macrophage Polarization: A Mathematical Model Frontiers*. Dec. 2022.